

PROBABILISTIC SSME BLADES STRUCTURAL RESPONSE
UNDER RANDOM PULSE LOADING

Michael Shiao, Robert Rubinstein, and Vinod K. Nagpal
Sverdrup Technology, Inc.
Lewis Research Center Group
Cleveland, Ohio

The purpose of this work is to develop models of random impacts on a Space Shuttle Main Engine (SSME) turbopump blade and to predict the probabilistic structural response of the blade to these impacts. The random loading is caused by the impact of debris. The probabilistic structural response is characterized by distribution functions for stress and displacements as functions of the loading parameters which determine the random pulse model. These parameters include pulse arrival, amplitude, and location. The analysis can be extended to predict level crossing rates. This requires knowledge of the joint distribution of the response and its derivative.

The model of random impacts chosen allows the pulse arrivals, pulse amplitudes, and pulse locations to be random. Specifically, the pulse arrivals are assumed to be governed by a Poisson process, which is characterized by a mean arrival rate. The pulse intensity is modelled as a normally distributed random variable with a zero mean chosen independently at each arrival. The standard deviation of the distribution is a measure of pulse intensity. Several different models were used for the pulse locations. For example, three points near the blade tip were chosen at which pulses were allowed to arrive with equal probability. Again, the locations were chosen independently at each arrival.

The structural response was analyzed both by direct Monte Carlo simulation and by a semi-analytical method. In the Monte Carlo method, appropriate random number generators were used to develop simulated pulse arrival processes. These processes were used as forcing functions in a dynamic analysis of the SSME blade implemented by the computer code STAEBL (Structural Tailoring of Engine Blades). The dynamic analysis originally used by this program was a modal superposition based on up to five modes; this analysis was replaced by a direct time integration which used the Newmark Beta Algorithm.

In the semi-analytical method, the classic analysis of shot noise by S. O. Rice was generalized to the present problem. This analysis requires that the unit pulse response of the blade be known at each point where a pulse can arrive. The unit response was found by numerical simulation using the STAEBL code. Once the unit pulse response is known, all required distribution functions are developed analytically.

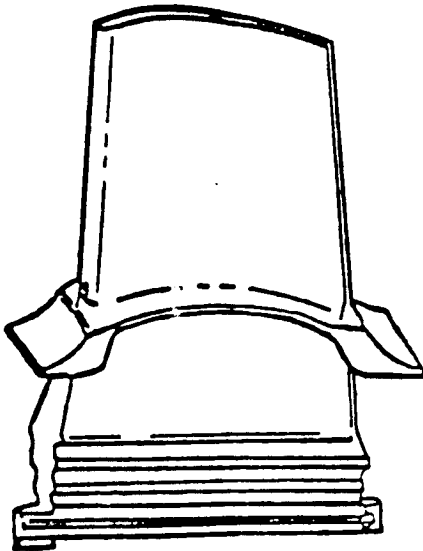
Comparisons between the Monte Carlo studies and the semi-analytical method showed excellent agreement. Of course, the semi-analytical method has the advantage of requiring considerably less computer time to implement.

An analysis was begun of the joint distribution of response and its time derivative. This joint distribution can be used to predict level crossing rates which are frequently used in fatigue life predictions. The joint distributions were evaluated both by direct simulation and semi-analytically. In this case, the Monte Carlo simulations require very long integration times to produce smooth distributions.

Possible extensions of this work include clustering effects in level crossings, direct simulation of the level crossing process, and analysis of the distribution of extreme values.

OBJECTIVE

Estimate the influence of random loading on
SSME blade responses



SSME BLADE

RANDOM LOADINGS

1. Pulse
2. Pressure
3. Temperature
4. Centrifugal

RANDOM IMPULSE LOADING

- o Poisson arrival pattern
- o Random impulse amplitude
- o Random impulse location

RANDOM STRUCTURAL RESPONSES

- o Natural Frequency
- o Root Stress
- o Tip Displacement

SUBJECTED TO RANDOM PULSE LOADING

● Three Response Models

$$(1) \quad Y_1(t) = \sum h(t-t_k)$$

$h(t)$ = Unit Impulse Response

t_k = A Poisson Process

$$(2) \quad Y_2(t) = \sum a_k h(t-t_k)$$

a_k = Random Amplitude

$$(3) \quad Y_3(t) = \sum a_k h_k(t-t_k)$$

$h_k(t)$ = Random Unit Impulse Response
(Random Pulse Location)

Probabilistic characteristics of model 1
and 2 are partially known

● Model 1

$$Y(t) = \sum h(t-t_k)$$

● Probability Density Function of $Y(t)$

$$f_Y(y) = e^{-\lambda T} \sum_{k=0}^{\infty} \frac{g_k(y) (\lambda T)^k}{k!}$$

● Joint Probability Density Function of $Y(t)$ and $\dot{Y}(t)$

$$f_{Y\dot{Y}}(y, \dot{y}) = e^{-\lambda T} \sum_{k=0}^{\infty} \frac{g_k^*(y, \dot{y}) (\lambda T)^k}{k!}$$

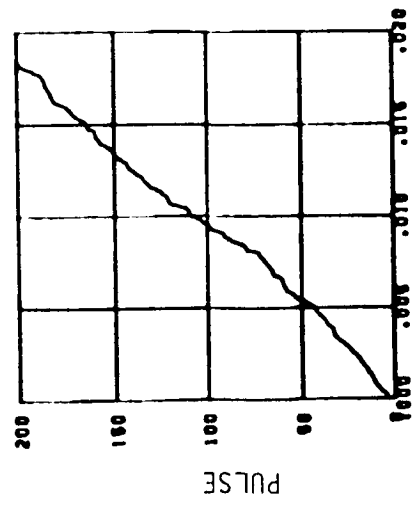
where λ = Poisson Arrival Rate

T = Unit Impulse Response Duration

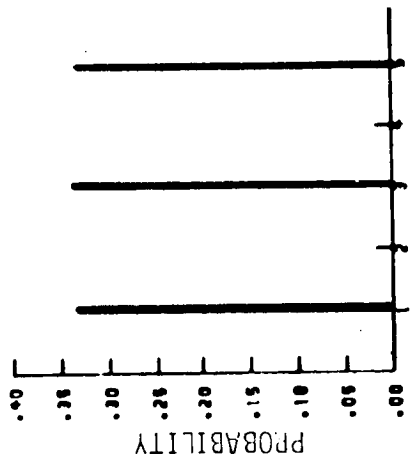
$g_k(y)$ = k-Fold Convolution of Probability Density
function of $h(t)$

$g_k^*(y, \dot{y})$ = k-Fold Convolution of Joint Probability
density function of $h(t)$ and $\dot{h}(t)$

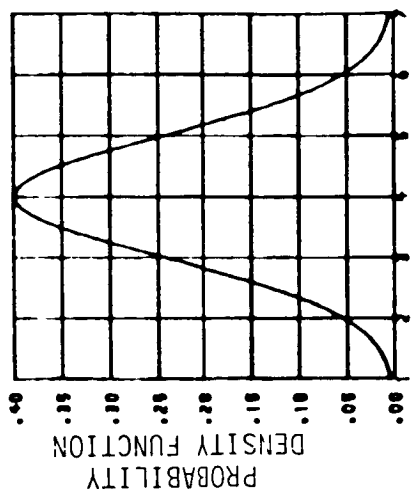
POISSON PROCESS



UNIFORM DISTRIBUTION



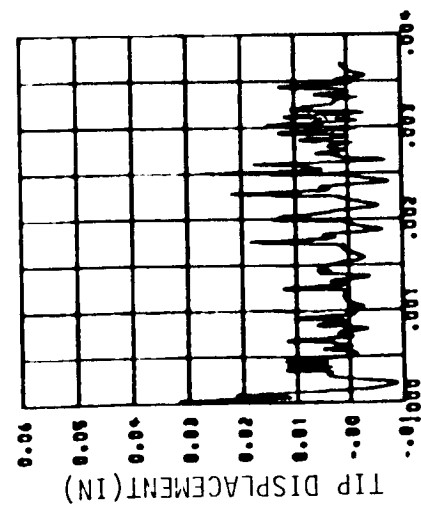
NORMAL DISTRIBUTION



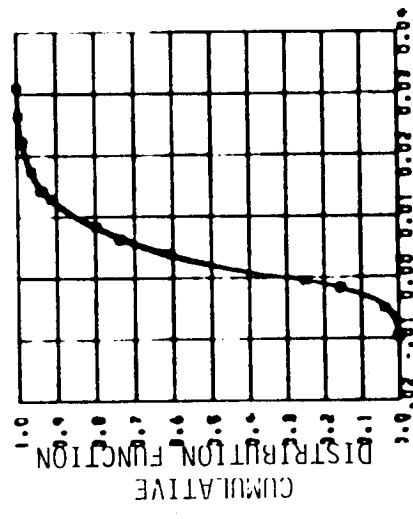
PULSE OCCURRENCE TIME (SEC)

PULSE LOCATION

PULSE MAGNITUDE (10^{-4})



TIME (SEC)



TIP DISPLACEMENT (IN)

POISSON MEAN ARRIVAL RATE

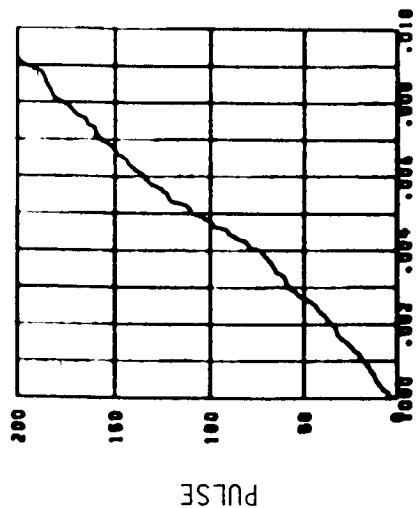
= 5000/sec

o -- Semi-Analytical Result

□ -- Simulation Result

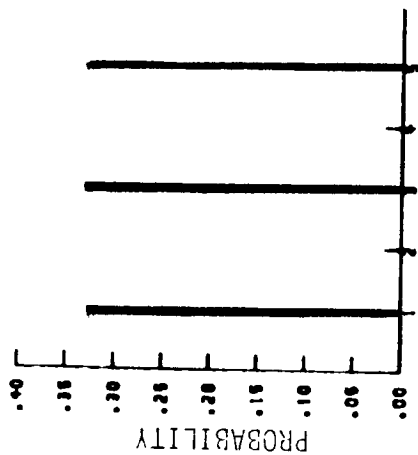
EXAMPLE 1 (MODEL 3)

POISSON PROCESS



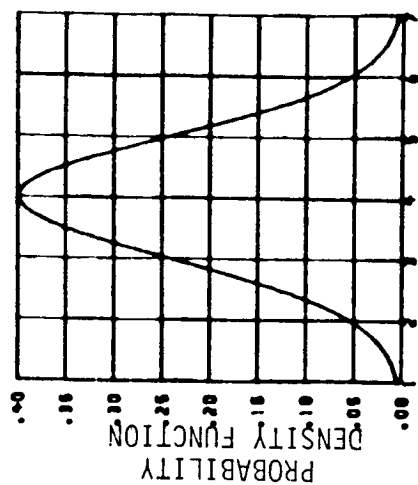
PULSE OCCURRENCE TIME (SEC)

UNIFORM DISTRIBUTION

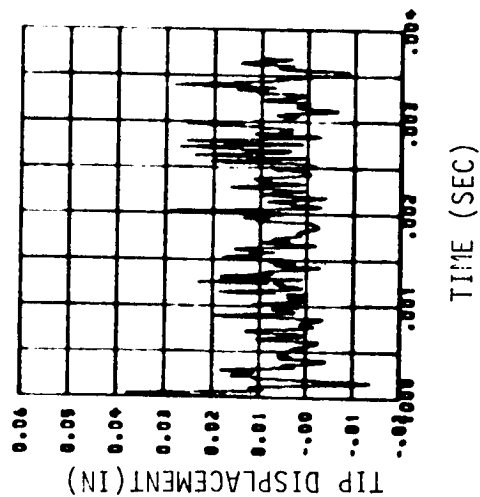


PULSE LOCATION

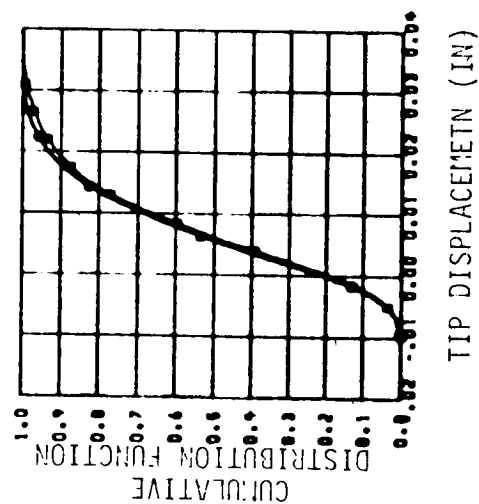
NORMAL DISTRIBUTION



PULSE MAGNITUDE (10^{-4})



TIME (SEC)



TIP DISPLACEMENT (IN)

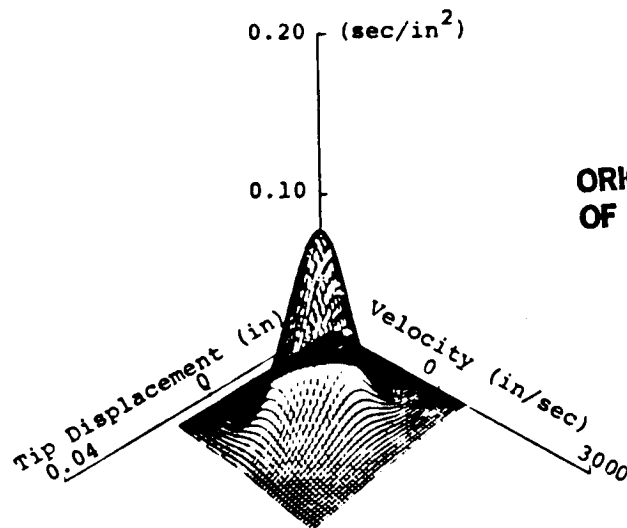
POISSON MEAN ARRIVAL RATE

= 10000/sec

o --Semi-Analytical Result

□ --Simulation Result

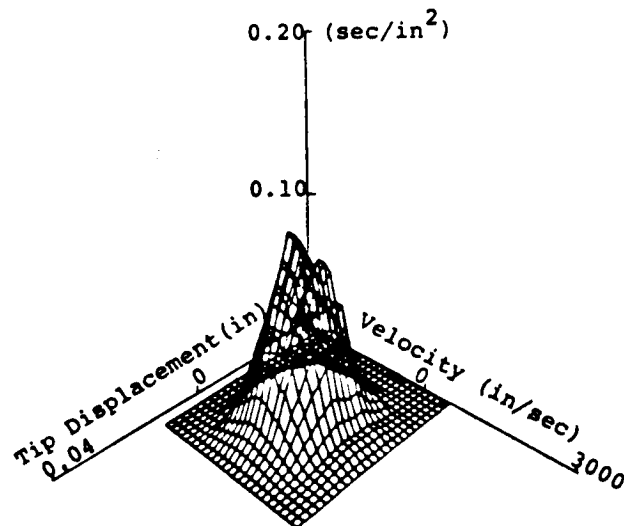
Joint Probability Density Function



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Theoretical Joint
Probability Density Function

Joint Probability Density Function



Simulated Joint
Probability Density Function

EXAMPLE 3 (MODEL 3)